

S. S. DEMPO COLLEGE OF COMMERCE AND ECONOMICS
CUJIRA, ST. CRUZ – GOA
Semester II Supplementary Examination, June – 2017
MATHEMATICAL TECHNIQUES
(Revised Course 2014-15 onwards)

Duration: 2 Hours

Marks: 80

- Instructions:**
1. All questions are compulsory.
 2. Figures to the right indicate full marks.
 3. Graph papers are provided with request.
 4. Scientific Calculator is not allowed.

Q. 1. Answer the following.

(5X4=20)

- a) If $f(x) = x^2 + 5x - 7$, $3 \leq x \leq 8$, find $f(1)$, $f(3)$, $f(5)$ and $f(7)$ if they exist. Find x if $f(x) = 7$.
- b) Find $\frac{dy}{dx}$ where i) $y = 7x^3 - 4 \log x + 2e^x - 10$
ii) $y = (x^5 - 5x)(x^6 - 2)$
- c) Prove that the points A(7, 9), B(3, -7) and C(-3, 3) are the vertices of a right angled triangle.
- d) Find the compound interest on ₹ 15,000 for 6 years at 8 % p.a. compounded half yearly. [Given that $(1.04)^{12} = 1.6010$]

OR

Q. I. Answer the following.

(5X4=20)

- w) If $f(x) = 3x - 5$, $1 \leq x \leq 6$, what is the domain and range of f ?
- x) Find $\frac{dy}{dx}$ where $y = (x^3 - 4x + 5)^6 + \log(x + \sqrt{x^2 + 5})$.
- y) In what ratio does the y - axis divide the join of A(3, 5) and B(6, 7)? Is the division internal or external?
- z) What is the effective rate of interest if the nominal rate is 5% p.a. compounded half-yearly?

Q. 2. Answer the following.

(5X4=20)

- a) Find the equation of a line through (2, -1) and parallel to the line $2x + 3y + 7 = 0$.
- b) The cost of producing x items is given by $C = x^2 + 4x + 4$. Find average cost and marginal cost when $x = 10$.
- c) Integrate the following with respect to x .

$$\text{i) } \int \left(6x^5 - 7e^x + \frac{5}{x^3} - 15 \right) dx$$

$$\text{ii) } \int \left(\frac{x^4 - 2x^3 + 5x + 3}{x^2} \right) dx$$

- d) Evaluate the following limits.

$$\text{i) } \lim_{x \rightarrow 5} \frac{\sqrt{x+4} - 3}{x-5} \quad \text{ii) } \lim_{x \rightarrow 2} \left[\frac{1}{x-2} - \frac{1}{x^2 - 3x + 2} \right]$$

OR**Q. II. Answer the following.**

(5X4=20)

- w) Find the equation of a line having y -intercept -3 and perpendicular to $x - 2y + 15 = 0$.
- x) Of the two given functions, $x = 50 - 4p - p^2$ and $x = 3p + 6$ state, with reasons, which function is the demand function and which is the supply function. Find the equilibrium price.

y) Evaluate $\int_{-1}^1 (3x^2 + 2kx + c) dx = 4$, find c .

- z) Examine the continuity of f at $x = 1$, if

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{when } x \neq 1 \\ 4 & \text{when } x = 1 \end{cases}$$

(5X4=20)

Q. 3. Answer the following.

- a) If $z = x^4 - 6x^3y^5 + y^2$, find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$, $\frac{\partial^2 z}{\partial x^2}$, $\frac{\partial^2 z}{\partial y^2}$.
- b) Find the amount of an annuity if payment of ₹ 100 is made annually for 5 years at interest rate of 7% p.a. compounded annually. [Given that $(1.07)^5 = 1.4026$]
- c) The demand function for the commodity is $p = 25 - 3x$, where p is a price per unit and x is the quantity demanded. Find consumer's surplus at price $p = 10$.
- d) Solve the following Linear Programming Problem by graphical method.

Maximize $z = 2x + 5y$
subject to $3x + y \leq 21$
 $x + y \leq 9$
 $x \geq 0, y \geq 0$

OR

(5X4=20)

Q. III. Answer the following.

- w) The production function is $P = x^2 - 2xy + y^2$, where x is labour and y is capital. Find the marginal physical products of labour and capital when $x = 2$ and $y = 1$.
- x) A man purchased a six years National Savings Certificate for ₹ 1000. After 6 years he got ₹ 2015. Find the rate of interest if the interest is compounded half-yearly.
[Given that $(2.015)^{\frac{1}{12}} = 1.0601$]
- y) The demand function for a commodity is $p = 24 - 6x$ and the supply function is $p = 2x + 8$, where x is the quantity supplied or demanded. Find producer's surplus at the equilibrium price.
- z) Solve the following Linear Programming Problem by graphical method.
- Minimize $z = 5x + 2y$
subject to $10x + 2y \geq 20$
 $5x + 5y \geq 30$
 $x \geq 0, y \geq 0$

(5X4=20)

Q. 4. Answer the following.

- a) P divides the segment AB internally in the ratio 3:2. If A (1, -2) and P (4, 7), find the co-ordinates of B.

- b) Find the output for the best profit of a firm where total revenue and total cost functions are given by $R = 20x - x^2$ and $C = x^2 + 8x + 2$.
- c) Find $\frac{dy}{dx}$ if $y = \frac{2x^3 - 5x + 6}{x^5 - 3}$
- d) If the marginal revenue of the firm is $MR = x^4 + 5x^2 + 12$, where x is the output, find total revenue function.

OR

Q. IV. Answer the following. (5X4=20)

- w) If A(3,-5) and B(1,3) are two points. Find the equation of the perpendicular bisector of segment AB.
- x) The demand function p in terms of quantity demanded is given by $p = 15 + 10x - 3x^2$.
Find average revenue and marginal revenue when the demand is 2 units.
- y) If the demand function is given by $x = 20 - 3p - p^2$, find the price elasticity of demand for $p = 3$.
- z) Evaluate the following integral.

i) $\int (4(2x+5)^3 + 7e^{3x-5}) dx$

ii) $\int \left(\sqrt{4x-3} - \frac{6}{3x+8} \right) dx$