

**S. S. DEMPO COLLEGE OF COMMERCE AND ECONOMICS**  
**CUJIRA, ST. CRUZ - GOA**  
**F.Y.B.Com. Semester II Examination, April - 2017**  
**MATHEMATICAL TECHNIQUES**  
**(Old Course)**

**Duration:** 2 Hours

**Marks:** 80

- Instructions:**
1. All questions are compulsory.
  2. Use of calculator is not allowed.
  3. Figures to the right indicate full marks.
  4. Graph paper and Log tables are provided with request.

**Q.1. Answer the following.**

(5X4=20)

- a) Find y if the distance between A(2, -3) and B(-1, y) is 5 units.
- b) Find  $\frac{dy}{dx}$  if
  - i)  $y = 3x^3 + \frac{2}{x^3} - 5e^x + 3\sqrt{x} + 15$ .
  - ii)  $y = 4e^{(x^3-3x+4)} + 6\log(2x-5)$
- c) The demand function p in terms of quantity demanded is given by  $p = 5x^2 - 3x + 10$ .  
 Find total revenue and marginal revenue when the demand is 5 units.
- d) If  $z = x^2 + 2xy - y^2$ , prove that  $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 2z$ .

**OR**

**Q.I. Answer the following.**

(5X4=20)

- w) Find the equation of a line having y-intercept -3 and parallel to  $3x - 2y + 10 = 0$ .
- x) Find  $\frac{dy}{dx}$  if  $y = \frac{3x-5}{x^4 e^x}$ .
- y) If the demand function is given by  $x = \frac{p-3}{p+1}$ , find the price elasticity of demand  
 for  $p = 4$ .

z) If  $z = x^3 - 10x^2y + y^3$ , show that  $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 3z$ .

**Q.2. Answer the following. (5X4=20)**

- a) If  $f(x) = 5 - 4x$ ,  $2 \leq x \leq 4$ , what is the domain and range of  $f$ ?
- b) Of the two given functions,  $x = 50 - 4p - p^2$  and  $x = 3p + 6$ , find out which function is the demand function and which is the supply function. Find the equilibrium price and the quantity demanded at that price.
- c) A function  $f$  is defined as

$$\begin{aligned} f(x) &= x^2 - 6 & 1 < x < 5 \\ &= 4x + 2 & 5 \leq x \leq 8 \end{aligned}$$

Discuss the continuity of  $f$  at  $x = 5$ .

- d) Find the equation of the line passing through the point of intersection of the lines  $x - 2y + 1 = 0$  and  $x + 2y + 1 = 0$  and perpendicular to  $2x - 3y + 10 = 0$ .

**OR**

**Q. II. Answer the following. (5X4=20)**

w) If  $f(x) = \frac{3x+5}{2x-3}$ , show that  $f(f(x)) = x$ .

x) The total cost function is given by  $C = x^2 + 15x + 200$ , where  $x$  is the output. Determine the average cost and marginal average cost when  $x = 20$ .

y) Find  $\lim_{x \rightarrow 2} \frac{x^3 - 2x^2 - 4x + 8}{x^3 - 5x^2 + 8x - 4}$ .

z) If A (3, -2), B (2, -1) and C (0, 7) are the vertices of a triangle ABC, find the equation of the side AB.

**Q.3. Answer the following. (5X4=20)**

- a) The marginal cost function of a firm is given by  $MC = 6x^3 + 9x^2 + 20$ , where  $x$  is the output. Find the total cost function if the fixed cost is ₹ 400.

- b) A sum of ₹ 75,000 is invested at 5% p.a. for 10 years. Find the compound interest if the interest is compounded quarterly.
- c) The demand function for a commodity is  $p = 15 - 5x$  and the supply function is  $p = 3x - 1$ . Find the consumer's surplus at the equilibrium price.
- d) The x-intercept of a line is double its y-intercept. If it passes through (1, -3), find its equation.

**OR**

**Q. III. Answer the following. (5X4=20)**

w)  $\int_{-1}^1 (x^2 + 1)(4x + 2) dx$

- x) Find the present value of ₹ 60,000 to be received after 5 years at 8% p.a. compounded half yearly.
- y) The supply function for a commodity is  $p = x^2 + 5x + 4$ , where x is a quantity supplied. Find the producer's surplus, when the price is 10.
- z) In what ratio is the line joining A(7, 8) and B(-5, -1) divided by y-axis? Is the division internal or external?

**Q.4. Answer the following. (5X4=20)**

- a) Integrate the following with respect to x.

i)  $\int (x^7 - \frac{4}{x^3} + \frac{3}{x} + 5e^x + 2\sqrt{x}) dx$

ii)  $\int \left( \frac{2x^4 - 7x^3 + 6x^2}{x^2 - 2x} \right) dx$

- b) A firm produces an output of x tons at a total cost  $C = x^3 - 4x^2 + 7x$ . Find the output at which the average cost is minimum.
- c) Find the amount of an annuity of ₹ 40,000 payable at the end of each year for 6 years at 8% p.a. compounded quarterly.

d) Solve the following linear programming problem graphically.

$$\text{Minimize } z = 12x + 20y$$

$$\text{subject to } x + y \geq 7$$

$$5x + 2y \geq 20$$

$$x \geq 0, y \geq 0$$

OR

Q. IV. Answer the following.

(5X4=20)

w) Evaluate the following integral.

$$\int \left( 2(4x+5)^3 + 7^{2x-3} - \frac{6}{5-2x} + e^{3x-6} \right) dx$$

x) Find  $\frac{dy}{dx}$  if  $y = 5x^2 \log x - (x^4 - 4)7^x$

y) What is the effective rate of interest if the nominal rate is 6 % p.a. compounded quarterly?

z) Solve the following linear programming problem graphically.

$$\text{Maximize } z = 300x + 200y$$

$$\text{subject to } 5x + 2y \leq 180$$

$$x + y \leq 45$$

$$x \geq 0, y \geq 0$$