

S. S. DEMPO COLLEGE OF COMMERCE AND ECONOMICS
CUJIRA, ST. CRUZ - GOA
F.Y.B.Com. Semester II Examination, April - 2017
MATHEMATICAL TECHNIQUES
(Revised Course 2014-15 onwards)

Duration: 2 Hours

Marks: 80

- Instructions:**
1. *All questions are compulsory.*
 2. *Figures to the right indicate full marks.*
 3. *Graph papers are provided with request.*
 4. *Scientific Calculator is not allowed.*

Q.1. Answer the following.

(5X4=20)

- a) Find y if the distance between A(2, -3) and B(-1, y) is 5 units.
- b) Find $\frac{dy}{dx}$ if
 - i) $y = 3x^3 + \frac{2}{x^3} - 5e^x + 3\sqrt{x} + 15.$
 - ii) $y = 4e^{(x^3-3x+4)} + 6\log(2x-5)$
- c) The demand function p in terms of quantity demanded is given by $p = 5x^2 - 3x + 10.$
Find total revenue and marginal revenue when the demand is 5 units.
- d) If $z = x^2 + 2xy - y^2,$ prove that $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 2z.$

OR

Q.I. Answer the following.

(5X4=20)

- w) Find the equation of a line having y-intercept -3 and parallel to $3x - 2y + 10 = 0.$
- x) Find $\frac{dy}{dx}$ if $y = \frac{3x-5}{x^4 e^x}.$
- y) If the demand function is given by $x = \frac{p-3}{p+1},$ find the price elasticity of demand for $p = 4.$

z) If $z = x^3 - 10x^2y + y^3$, show that $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 3z$.

Q.2. Answer the following.

(5X4=20)

- a) If $f(x) = x^2 + 5x - 3$ and $g(x) = 2x + 1$, find $f(g(x))$ and $g(f(x))$.
- b) Of the two given functions, $x = 50 - 4p - p^2$ and $x = 3p + 6$, find out which function is the demand function and which is the supply function. Find the equilibrium price and the quantity demanded at that price.
- c) A function f is defined as.

$$f(x) = \begin{cases} x^2 - 6 & 1 < x < 5 \\ 4x + 2 & 5 \leq x \leq 8 \end{cases}$$

Discuss the continuity of f at $x = 5$.

- d) Find the equation of the line passing through the point of intersection of the lines $x - 2y + 1 = 0$ and $x + 2y + 1 = 0$ and perpendicular to $2x - 3y + 10 = 0$.

OR

Q. II. Answer the following.

(5X4=20)

w) If $f(x) = \frac{3x+5}{2x-3}$, show that $f(f(x)) = x$.

- x) The total cost function is given by $C = x^2 + 15x + 200$, where x is the output. Determine the average cost and marginal average cost when $x = 20$.

y) Find $\lim_{x \rightarrow 3} \frac{\sqrt{x^2+7}-4}{x^2-5x+6}$

- z) If $A(3, -5)$ and $B(1, 3)$ are two points. Find the equation of the perpendicular bisector of segment AB .

Q.3. Answer the following.

(5X4=20)

- a) The marginal cost function of a firm is given by $MC = 6x^3 + 9x^2 + 20$, where x is the output. Find the total cost function if the fixed cost is ₹ 400.

- b) A sum of ₹ 75,000 is invested at 5% p.a. for 10 years. Find the compound interest if the interest is compounded quarterly. [Given that $(1.0125)^{40} = 1.6436$]
- c) The demand function for a commodity is $p = 15 - 5x$ and the supply function is $p = 3x - 1$. Find the consumer's surplus at the equilibrium price.
- d) The vertices of a triangle ABC are A(2, 0), B(2, -4) and C(1, -2). Find the equation of the altitude from A.

OR

Q. III. Answer the following.

(5X4=20)

w) $\int_{-1}^1 (x^2 + 1)(4x + 2) dx$

- x) Find the present value of ₹ 60,000 to be received after 5 years at 8% p.a. compounded half yearly. [Given that $(1.04)^{10} = 1.4802$]
- y) The supply function for a commodity is $p = x^2 + 5x + 4$, where x is a quantity supplied. Find the producer's surplus, when the price is 10.
- z) In what ratio is the line joining A(7, 8) and B(-5, -1) divided by y-axis? Is the division internal or external?

Q.4. Answer the following.

(5X4=20)

a) Integrate the following with respect to x.

i) $\int (x^7 - \frac{4}{x^3} + \frac{3}{x} + 5e^x + 2\sqrt{x}) dx$

ii) $\int \left(\frac{2x^4 - 7x^3 + 6x^2}{x^2 - 2x} \right) dx$

- b) A firm produces an output of x tons at a total cost $C = x^3 - 4x^2 + 7x$. Find the output at which the average cost is minimum.
- c) Find the amount of an annuity of ₹ 40,000 payable at the end of each year for 6 years at 8% p.a. compounded quarterly. [Given that $(1.02)^{24} = 1.6084$]

d) Solve the following linear programming problem graphically.

$$\text{Minimize } z = 12x + 20y$$

$$\text{subject to } x + y \geq 7$$

$$5x + 2y \geq 20$$

$$x \geq 0, y \geq 0$$

OR

Q. IV. Answer the following.

(5X4=20)

w) Evaluate the following integral.

$$\int \left(2(4x+5)^3 + 7^{2x-3} - \frac{6}{5-2x} + e^{3x-6} \right) dx$$

x) Find $\frac{dy}{dx}$ if $y = 5x^2 \log x - (x^4 - 4)7^x$

y) What is the effective rate of interest if the nominal rate is 6 % p.a. compounded quarterly?

z) Solve the following linear programming problem graphically.

$$\text{Maximize } z = 300x + 200y$$

$$\text{subject to } 5x + 2y \leq 180$$

$$x + y \leq 45$$

$$x \geq 0, y \geq 0$$